Stability Response of the Stochastic, Delay Version of the Duffing Eqaution

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In the past three decades, the study of stability of buildings, bridges, beams, columns or shells under the influence of excitations has traditionally modeled central deflections by the most celebrated Duffing equation with emphasis on stochastic effects to yield equation of the form

$$\ddot{x} + 2[\zeta + f(t)]\dot{x} + [(\gamma + g(t)]x + \beta_3 x^3 = 0,$$

where the real numbers ζ, γ, β_3 are ζ, γ are the damping, stiffness and nonlinear coefficients, respectively, and $f(t)$, $g(t)$ are zero-mean ergodic stochastic processes with smoothly varying spectral density functions and small correlation time. The study of stability of solutions of this stochastic version of the Duffing equation when one or both of $f(t)$ and $g(t)$ are present is well established in the literature. Time delay appears to be a natural occurrence in structural systems as a result of the excitation of feedback mechanisms, however, less attention is being paid on the instability induced by time delays. In this paper, the influence of the stochastci delay version of the Duffing equation is studied. A linearized stability analysis of a transcendental characteristic equation of the Duffung equation is analyzed. As stability is lost, two types of bifurcations, namely, subcritical and supercritical bifurcations are discussed. Then, conditions ensuring stable and unstable behaviour due to linear stochastic perturbation are derived. The Markovian diffusion approximation according to the integral averaging method and the Lyapunov exponent are employed to obtain explicit analytical results relating to the stability conditions in the stochastic sense. Derived analytical results are quantified numerically.