A Novel Method for Reducing the Complexity of Fluid-Structure Problems

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Understanding the physics of fluid-solid problems is very challenging as it involves two inherently infinite dimensional phenomena. The fluid equations and solid equations are both governed by a set of coupled partial differential equations. Since very little progress can be done from an analytic point of view one approach is to understand these complicated coupled equations through numerical simulations. Central to many numerical simulations is the problem of representing a given partial differential equation by finite set of ordinary differential equations. This process is achieved through the use of some form of projection technique. However even these finite number of retained modes, of the order of few millions, is very large. It is of considerable interest to project the dynamics of these large number of ordinary differential equations onto a proper low dimensional subspace on which most of the relevant important dynamics evolve.

The traditional methods used in fluid mechanics are Karhunen-Loeve decomposition or Principal orthogonal decomposition (POD) and Singular perturbation technique. The main idea in POD is projection of the dynamics of the system onto few basis functions which have most of the energy (optimal energy in the L_2 sense). Singular perturbation is an time scale separation technique, which projects the dynamics onto a slow manifold by

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truncating the fast manifold dynamics.

Even though for some applications the most energetic modes are the important modes, this need not be the case always. The important thing in any problem is, what is driving the system (input), and what is it that one is interested in (output). We will argue that capturing this input-output behavour is very important in truncating modes and getting simple models. In this paper we introduce a complexity reduction technique for fluids-structure problems - borrowing ideas from control theory - that takes into account the underlying input-output properties of fluids. This method has considerable advantages like rigorous error bounds, transparent physics and preservation of inter connection. The error is quantified in terms of the H_∞ norm of the difference of the full and truncated transfer functions. Preservation of inter connection is a very important for fluid-structure interaction problems, as we do not want the solid model to be changed when ever the fluid model has been changed and vice-versa. This will save lot of computational time and memory. The physics becomes clear through the use of the new concepts like controllability and observability operators.

Controllability operator tells about the modes that are reachable or unreachable with a given input. The reachable modes are called the controllable modes. Hence, unreachable modes are are not influenced by the input. Observability operator tells about the modes that influence the output. These modes are called observable modes. Therefore, unobservable modes do not have any influence on the output. The main idea behind this method is deleting the weakly controllable and weakly observable modes of the system after their respective ellipsoids are aligned through a similarity transformation. The relative importance of a state in the input-output behavior of the system is given by the corresponding Hankel singular value.

To illustrate the above ideas we apply the above methodology to a simple fluid-structure problem: oscillatory motion of an elastic sphere in a fluid. Approximating the sphere as an elastic spring with one degree of freedom, and considering traveling wave solutions for pressure in fluid, one can reduce the coupled system into a initial value problem with three states. We took the input to be some disturbance in the fluid and output to be the radius of the sphere. The Hankel singular values for this system are found to be $(0.4360, 0.1934, 0.0074)$. It can be seen that the first two singular values are of the same order and the third singular value is an order of magnitude smaller than the first two. This implies that the third mode has little influence on the input-output properties of the system. Figure(1) shows the plot of radius as a function of time with a delta function as input in the full and truncated (with last two modes truncated) problems. We can see from the plot that though the steady state behavior is captured well, the initial transient is not captured well by the truncated system. Figure(2) shows the plot of radius as a function of time with a Heaviside function as input in the full and truncated (with last two modes truncated) problems. The truncated model is way of from the full model in this case. The poor performance of the above truncated model is due to the fact that we truncated a second mode which is important. Figure(3) and Figure(4) are the the same plots as in Figure(1) and Figure(2) respectively, but with only last one mode truncated. One can see that the agreement between the full and truncated model is good now.

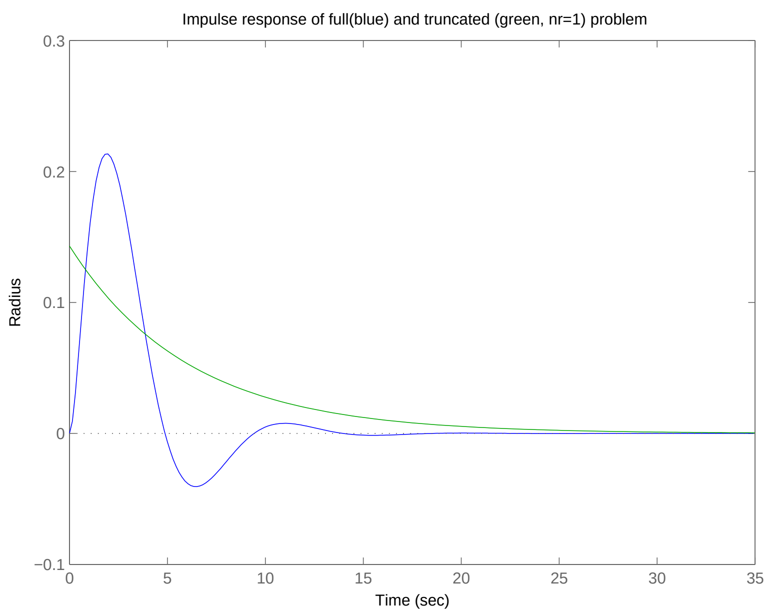

Figure 1: Delta response of full and truncated (nr=1) models

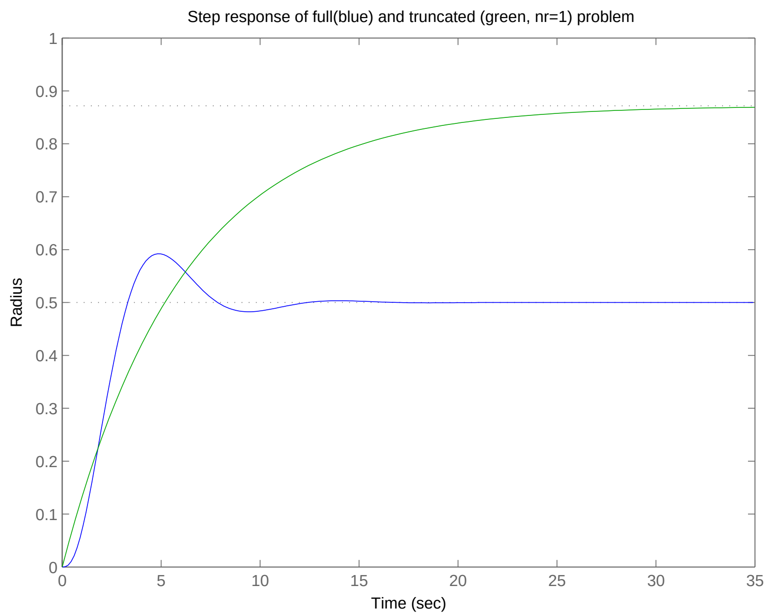

Figure 2: Step response of full and truncated (nr=1) models

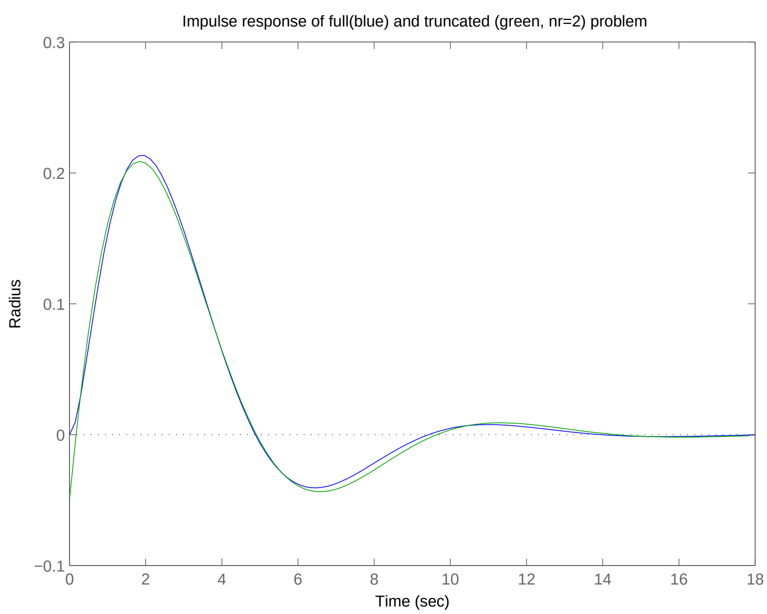

Figure 3: Delta response of full and truncated (nr=2) models

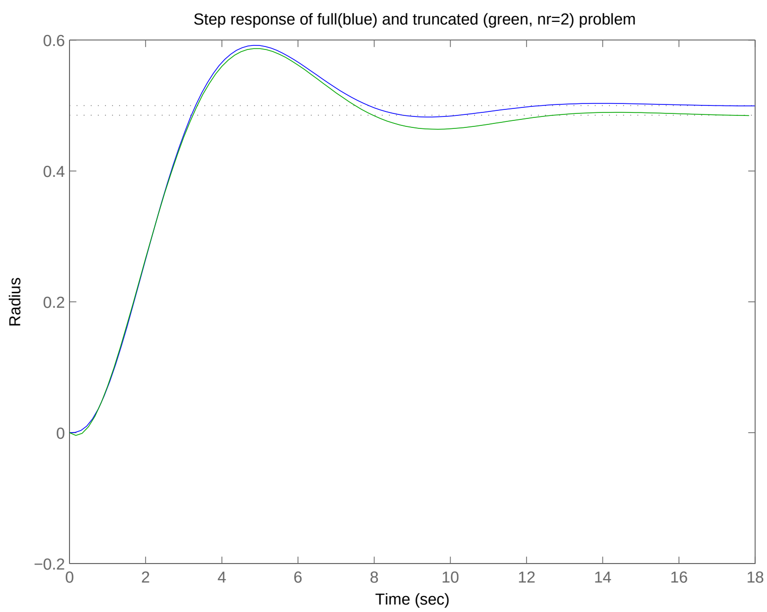

Figure 4: Step response of full and truncated (nr=2) models