Dynamic Stability of Stochastic Delay Systems

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This paper is aimed at examining the influence of time delays and multiplicative white noise on nonlinear dynamical system that exhibits Degenerate Hopf interactions. Our desire in such a system stems from the continued interest to understand the stability behaviour of mechanical systems subjected to fluctuating regenerative excitations. The excitations introduce time delays in the displacement functions, damping and restoring forces, and may produce persistent modes of system's performance failures even at incipient time delays prior to the fully growth rate of the excitations. Systems involving time delays are best described by delay differential equations, and analysis to such equations are accompanied with complications, even when nonlinearities are omitted. The consideration of multiplicative white noise adds a greater impediment to the analysis. Frequently in many investigations, the assumption of small time delay as compared to unity is being imposed in order to reduce the infinite-dimensional character of the time problem to a finite-dimensional problem using conventional techniques such as (i) the Taylor Series Expansion, (ii) the Integral Averaging Method, (iii) Fourier Series, (iv) Multiple Scale and (vi) Harmonic Balancing, and then examine the stability behaviour of the reduced systems. It is well known that the use of these asymptotic techniques with the assumption of small time delays usually lends itself to results, which do not reflect the long term stability behaviour of the original time delay system. We focus here on the use of the centre manifold theorem, properties of semigroup of infinitesimal generators and the classical theorem of Hopf bifurcation for the study of periodic solutions of dynamical systems, with special attention that the long term stability behaviour of the original nonlinear stochastic delay system is preserved, and moreover the time delays are not small. The mechanical system considered is modelled as a single degree of freedom and the equations governing the motion contain multiplicative white noise, multiple time delays and nonlinearity. The computation of the Poincaré-Lyapunov coefficients, Floquet exponents and moment Lyapunov exponents of the reduced system will enable us to determine sufficient conditions for the possible degenerate Hopf interactions. The dependence of the interactions on parameter variations will be captured qualitatively.